



SWANSEA METROPOLITAN UNIVERSITY

PRIFYSGOL FETROPOLITAN ABERTAWE

EXAMINATIONS 2008 / 2009 AHOLIADAU 2008 / 2009

Semester 2

FACULTY OF: Applied Design & Engineering

NAME OF COURSE/PROGRAMME:

BEng Motorcycle Engineering

YEAR OR LEVEL:

Year 2

NAME OF MODULE/UNIT/EXAM PAPER:

**Motorcycle Dynamics and Power
Transmission**

MODULE NO:

DATE OF EXAMINATION:

Wednesday 27th May 2009

EXAMINATION TIMES:

START TIME:

12.00 pm

FINISH TIME:

2.00 pm

TIME ALLOWED:

2 hours

INSTRUCTIONS TO CANDIDATES:

For all questions, take $\rho_{air} = 1.167 \text{ kg m}^{-3}$

An equation sheet is provided with this exam paper

Answer 3 from 4 Questions

Question 1 - (Steady State Cornering)

- a. Show with the aid of diagrams, that the required lean angle for a motorcycle with 'thin' tyres in steady state cornering is equivalent to:

$$\phi_i = \tan^{-1} \left(\frac{V^2}{R_c g} \right)$$

(6 marks)

- b. Derive, with the aid of diagrams, an equation for the required lean angle for a motorcycle with finite thickness tyres.

(6 marks)

- c. Considering the gyroscopic contributions from only the motorcycle wheels, determine the correct lean angle for a motorcycle in steady state cornering if it has the following properties:

Mass, $m = 256$ kg (including rider)

Height of centre of gravity, $h = 0.62$ m

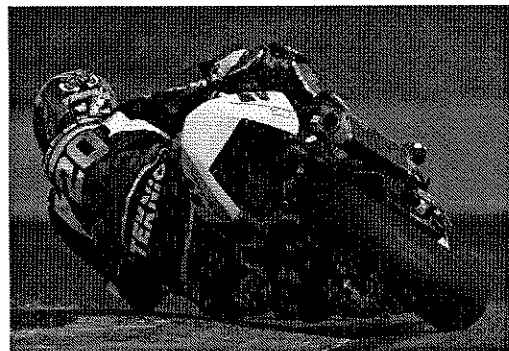
Wheel radius, $R_f = R_r = 0.3$ m

Inertia of wheels $I_{or} = I_{of} = 0.6$ kgm²

Tyre size = 180/70-17, $t = 70$ mm

Forward velocity, $v = 140$ kmh⁻¹

Corner Radius, $R = 135$ m



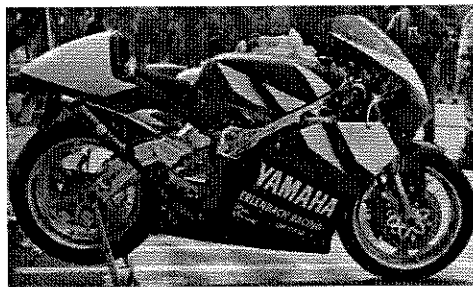
(8 marks)

Question 2 – (Suspension Analysis)

- a. A motorcycle suspension can be represented by an equivalent 'vertical stiffness' or 'wheel rate'. Using both diagrams and calculations, show how the 'wheel rate' and the 'spring rate' are related.

(8 marks)

- b. The Tech3/SMU Yamaha TZ250 weighs 126kg and has a 58/42 front biased weight distribution with rider. The fork travel is 110mm, the rear wheel travel is 114mm, the rake angle is 21.5° , the front unsprung mass is 11kg and the rear unsprung mass is 14.5kg.



Advise a suitable ride frequency assuming your own weight and the motorcycle, and from this determine a suitable fork spring stiffness.

(6 marks)

- c. The rear suspension can also be analysed in a similar manner. If the 'link ratio' for the motorcycle is 2.5:1 and the damping coefficient in compression at the damper is 1100 Ns/m, calculate the required stiffness of the rear spring and comment on the suitability of the damping coefficient and its corresponding value at the shock.

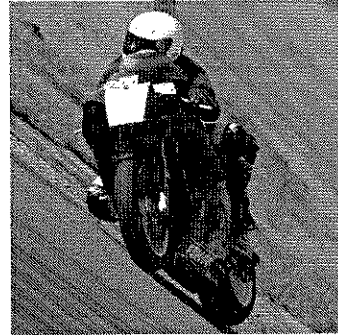
(6 marks)

Question 3 – (Rectilinear Dynamics)

- a. Derive a set of equations that can be used to determine the wheel loads of a motorcycle in both static and dynamic situations. (8 marks)

Consider a motorcycle with the following properties:

Mass with rider, $m = 245\text{kg}$
Reduced Mass, $m^* = 265\text{kg}$
Frontal Area, $A = 0.85\text{ m}^2$
Drag Coefficient, $C_D = 0.75$
Wheelbase, $p = 1.4\text{m}$
50/50 weight distribution
CofG height, $h = 0.6\text{m}$
Front fork stiffness $k_f = 22\text{ N/mm}$
Rake angle = 22°



- b. What is the *total* driving force required to maintain the motorcycle at a constant velocity of 185 kmh^{-1} along a horizontal flat road? (3 marks)
- c. If the motorcycle has a maximum power output of 125KW , what would you expect its aerodynamically limited top speed to be? (3 marks)
- d. For this motorcycle in braking, its data logger shows a deceleration of $0.8g$, with a suspension compression of 80mm . Using the data provided above, show using calculations that this is a reasonable suspension compression value.

(6 marks)

Question 4 – (Gyroscopic Effects)

- a. To turn a motorcycle at speed, a complex process is required. Describe the processes involved in instigating a turn to the right and then a steady state cornering procedure.

(6 marks)

- b. Consider a motorcycle with the following properties:

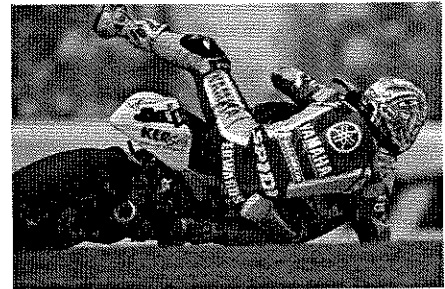
Moment of inertia, front wheel, $I_{wf} = 0.6 \text{ kgm}^2$

Spin velocity of wheel, $\omega = 100 \text{ rad s}^{-1}$

Moment of inertia, front frame, $I_{zf} = 0.45 \text{ kgm}^2$

Rake angle, $\varepsilon = 25^\circ$

Normal trail, $a_n = 0.1 \text{ m}$



- i. How much torque is required about the steer axis to effect a roll rate of 1 rad s^{-1} ?

(4 marks)

- ii. What effective 'steer rate' would result in a countersteered roll rate of 1 rad s^{-1} ?

(4 marks)

- iii. Assuming the engine is a transversely mounted 4 cylinder with the crank spin direction being opposite to the spin direction of the wheels, what effect do you think the engine will have on the roll rate of the motorcycle?

(6 marks)

Equation Data Sheet

Dynamic Loadings:

$$N_f = mg \frac{b}{p} - S \frac{h}{p}$$

$$N_R = mg \frac{(p-b)}{p} + S \frac{h}{p}$$

Aerodynamic Drag:

$$Force = \frac{1}{2} \rho C_D A V^2$$

Maximum Acceleration

$$\ddot{x} = \frac{mg}{m^*} \frac{b}{h} - \frac{F_D}{m^*}$$

Maximum Deceleration

$$\ddot{x} = -\frac{g(p-b)}{h} - \frac{F_D}{m}$$

Ideal Roll Angle

$$\varphi_i = \tan^{-1} \left(\frac{R_c \Omega^2}{g} \right)$$

Tyre Correction

$$\frac{\sin \Delta \varphi}{t} = \frac{\sin \varphi_i}{h-t}$$

Change in Roll Angle

$$\sin(\Delta \varphi) = \frac{I_w \omega \Omega \cos \varphi}{h \sqrt{(mg)^2 + (m R_c \Omega^2)^2}}$$

Gyroscopic Moment

$$M_g = I_w \omega \Omega \cos \varphi$$

$$M_g = d \sqrt{(mg)^2 + (m R_c \Omega^2)^2}$$

Gyroscopic Moment on Front Frame due to Roll Motion

$$M_g = I_{w_f} \omega_f \dot{\varphi} \cos \varepsilon$$

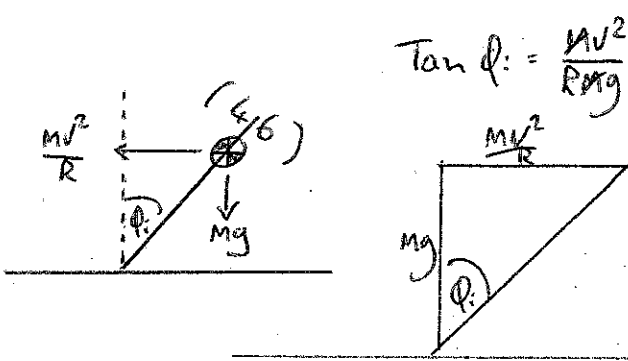
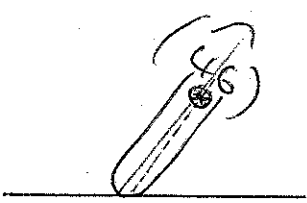
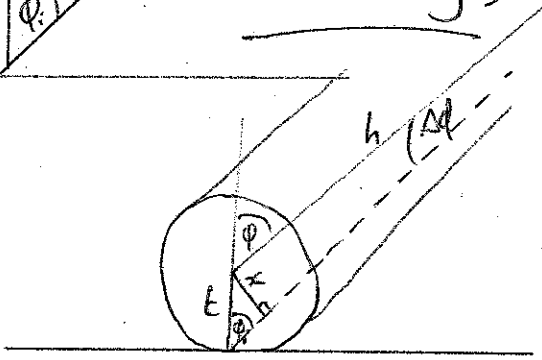
Natural Frequency

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{M}}$$



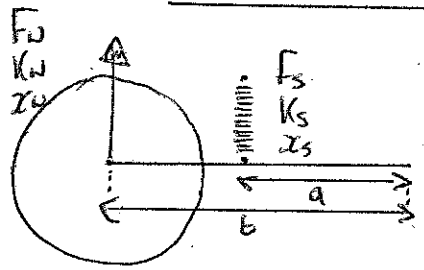
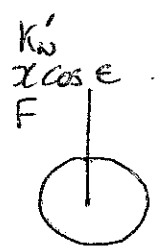
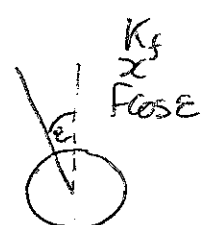
EXAMINATION SOLUTIONS : SESSION 2008/09

Programme:	BEng Motorcycle Engineering	Year:	Level 5
Module/Unit:	Motorcycle Dynamics and Power Transmission		
Author:	Dr Owen Williams	Page:	1 of 6

Question No	Question	Marks
1 a	 $\tan \phi = \frac{mv^2}{Rmg} = \frac{v^2}{Rg}$ $\phi = \tan^{-1}\left(\frac{v^2}{Rg}\right)$	6
b	  $\phi = \phi_i + \Delta\phi$ $\textcircled{1} \sin \phi_i = \frac{x}{t} \therefore x = t \sin \phi_i$ $\textcircled{2} \sin \Delta\phi = \frac{x}{h-t} \therefore x = (h-t) \sin \Delta\phi$ $t \sin \phi_i = (h-t) \sin \Delta\phi$ $\frac{\sin \Delta\phi}{t} = \frac{\sin \phi_i}{h-t}$	6

Continuation Sheet

Module/Unit:	Motorcycle Dynamics and Power Transmission	Page:	2 of 6
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Question No	Question	Marks
1c	$\phi_i = \tan^{-1} \left(\frac{v^2}{Rg} \right) = \tan^{-1} \left(\frac{38.8^2}{135g} \right) = 48.66^\circ$ $\Delta\phi = \sin^{-1} \left(\frac{t \sin \phi_i}{h-t} \right) = 5.48^\circ$ $\Delta\phi_g = \sin^{-1} \left(\frac{I_w \omega R \cos \phi_i}{h \sqrt{(mg)^2 + (m R \omega^2)^2}} \right) = 0.63^\circ$ $\phi_{\text{Real}} = \phi_i + \Delta\phi + \Delta\phi_g$ $\phi_{\text{Real}} = 54.77^\circ$	8
2a	 $F = Kx$ $F_w = \frac{a}{b} F_s$ $x_w = \frac{b}{a} x_s$ $K_w = \frac{F_w}{x_w} = \frac{\frac{a}{b} F_s}{\frac{b}{a} x_s} = \frac{F_s}{x_s} \left(\frac{a}{b} \right)^2 = K_s \left(\frac{a}{b} \right)^2$  $F = K'_w x \cos \epsilon$  $F \cos \epsilon = K_s x$ So $K'_w x \cos \epsilon \cos \epsilon = K_s x$ $K'_w = \frac{K_s}{\cos^2 \epsilon}$	4

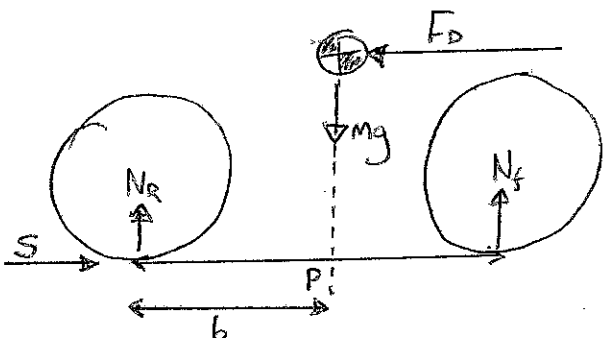
Continuation Sheet

Module/Unit:	Motorcycle Dynamics and Power Transmission	Page:	3 of 6
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Question No	Question	Marks
2b	Suitable Ride frequency - $2.0 \rightarrow 2.2 \text{ Hz}$ (2.2 Ideal) $f = \frac{1}{2\pi} \sqrt{\frac{K}{M}} \quad K = M(2\pi f)^2$ $K_w = 102.68 (2\pi \times 2.2)^2 = 19.62 \text{ N/mm}$ So $K_f = 19619.6 \cos^2(21.5) = 16.98 \text{ N/mm}$ So fork spring $K = \underline{8.5 \text{ N/mm}}$	6
2c	$f = \frac{1}{2\pi} \sqrt{\frac{K}{M}} \quad K_w = M(2\pi f)^2 = 67.82 (2\pi \times 2.2)^2$ $K_w = 12.96 \text{ N/mm}$ At shock $K_s = 12.96 \times 2.5^2$ $K_s = 81 \text{ N/mm}$ Damping at shock = 1100 Ns/m so at wheel = 176 Ns/m Critical damping = $2\sqrt{K_m}$ $= 2\sqrt{12.96 \times 67.82}$ $= 59.3 \text{ Ns/m}$ $\xi (\text{hela}) = \frac{C}{C_{\text{crit}}} = \frac{59.3}{176.0} = \underline{0.33}$ $\Rightarrow$ Content an underdamped ---	8 6
Question Total		20 Marks

Continuation Sheet

Module/Unit:	Motorcycle Dynamics and Power Transmission	Page:	4 of 6
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Question No	Question	Marks
3a	 $F_D = S$ ① $Mg = N_f + N_R$ ② Taking moments (CoG) $N_f(P-b) + Sh = N_R b$ Sub with ② for N_R $N_f(P-b) + Sh = Mg - N_f b$ $N_f P - \cancel{N_f b} + Sh = Mg - \cancel{N_f b}$ $N_f P + Sh = Mg b$ $N_f = Mg \frac{b}{P} - S \frac{h}{P}$ Now sub ② for N_f $(Mg - N_R)(P-b) + Sh = N_R b$ $Mg P - N_R P - Mg b + \cancel{N_R b} + Sh = \cancel{N_R b}$ $N_R P = Mg P - Mg b + Sh$ $N_R = Mg \left(\frac{P-b}{P} \right) + S \frac{h}{P}$ 6/ force = $\mu mg + \frac{1}{2} \rho C_D A V^2 = 0.02 \times 245g + \frac{1}{2} \rho \times 0.75 \times 0.85 \times \left(\frac{165 \times 5}{18} \right)^2$ force = <u>1030.4 N</u>	⑧ ⑤

Continuation Sheet

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Question No	Question	Marks
3c	$\text{Power} = \frac{1}{2} \rho C_D A V^3 \therefore V^3 = \frac{125000}{0.5 \rho C_D A} = 69.52 \text{ m/s}$ $\text{or } 250 \text{ km/h}$	(3)
3d	$N_f = mg \frac{P}{b} + S \frac{h}{P} \text{ for braking situations.}$ $N_f = 245g \left(\frac{0.7}{1.4} \right) + 0.8mg \left(\frac{0.6}{1.4} \right)$ $N_f = 122.5g + 84g = 206.5g$ Now $F = kx$ $206g = \left(\frac{22,000}{\cos^2 \epsilon} \right) x$ $x = \frac{206.5g \cos^2 \epsilon}{22,000}$ $x = \underline{79.1 \text{ mm}}$	

Continuation Sheet

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Question No	Question	Marks
4a	Description to include gyroscopic reactions influence on Roll and roll rate, with description of gyro effects in steady state cornering.	(6)
b	Torque reacted by gyro moment so $\approx$ $M_g = I_w \omega \dot{\phi} \cos E$ $= 0.6 \times 100 \times 1 \times \cos 25$ $= 54 \text{ Nm}$	(4)
c	Using this torque and reacting about the steer axis we get $M_g = I_{zf} \dot{\delta} \dot{\phi} \cos e$ $54 = 0.45 \dot{\delta} \times 1 \times \cos 25$ $\dot{\delta} = 2.45 \text{ rad/s}$ This does not physically result in a steer rate as in real life the gyroscopic forces are canceled by roll moment due to gyroscopic conservation of momentum	(4)
d	Reverse springing action reduces gyro moment but effect is very small compared to effect from wheels $\Rightarrow$ Small increase in steer/roll response but very little difference to yaw/roll in steady state	(6)
Question Total		20 marks